Multi-level 3-D Rotational Invariant Classification

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Abstract

A two-level 3-D rotational invariant classification is developed based on Fractional Differencing model. In first level, classification has been done with a fractal scale, and in second level, textures have been classified further in detail with the additional frequency parameters. Because of the properties of the fractal scale and multi-level procedure, the proposed 3-D rotational invariant classification scheme reduces the processing time and gives enough accuracy of the classification simultaneously. As a result of a series of experiments involving the differently oriented samples of natural textures, it is concluded that these combined features make possible for this multi-level classification method to have a strong class separability power for arbitrary oriented 3-D texture patterns.

1. Introduction

Texture classification has been the focus of interest to many researchers [1,2,3,4]. The classification problem can be stated as allocation of an observed texture image data to the one of the pre-defined texture classes. These texture classes can be described by texture features, and then texture features can be the parameters in stochastic [1,2,3] or structural models [4]. Thus, the key step in classification process is the choice of a set of features which can reduce the dimension of the image data to a computationally reasonable amount of data. Preferable features should be those that are simple and easy to extract from the given data while preserving the classifying information present in it.

In this paper, a multi-level classification method which can handle arbitrary 3-D rotated samples of textures is developed based on fractional differencing models with a fractal scaling parameter. Since the fractal scale is known to be a rotational and scaling invariant parameter, the accuracy of classification will not be affected by 3-D rotation of the test texture, by using these models. In first level of classification, the textures are classified by the first-order Fractional Differencing model with a fractal scale parameter, and in second level, classification is completed with the additional frequency parameters of the second-order Fractional Differencing periodic model. This multi-level classification scheme has at least following advantages over the conventional approaches [1,4,5]. First, since the fractal scale parameter of the first-order Fractional Differencing model can be estimated by a simple Least-square estimation method, the processing time can be dramatically reduced. Second, the ambiguity, which may be caused by using only one classification parameter, particularly when the values of parameter are close enough for the different textures, can be removed by considering the additional parameters in second-order model. Even though the estimation scheme for these additional parameters is based on ML estimation, the classification process is much simpler compared with other approaches based on ML estimation, e.g. Cohen's [1], because only a small number of texture patterns in same sub-class, which is already classified based on the fractal scale value in first level, need to be classified.

2. Fractional Differencing Model With One Fractal Scaling Parameter c.

Among the various random field models, the fractional differencing models with one fractal scaling parameter are chosen in this study because of its following properties. First, the fractal scaling parameter c is known to be a rotational and scaling invariant parameter, thus, with this parameter we have the flexibility to handle the rotated, slanted, and tilted texture surface. Second, unlike the other fractal model based on the fractional Brownian process, this model has a simple estimation scheme, such as least mean square estimation for the parameter c. Third, with the second order
periodic model, this model has good performance in texture synthesis and its ability to simultaneously represent the coarseness and pattern of texture surface with the fractal scaling parameter \( c \) and directional frequency parameters \( \omega_1, \omega_2 \), respectively. Thus, comparing these parameter values, we can classify the texture patterns properly even though some texture patterns share the same value of one of those parameters. Typical first-order and second-order fractional differencing models with one fractal scaling parameter will be as follows, respectively.

\[
y(m_1, m_2) = [(1-z_1^{-1})(1-z_2^{-1})]^{-\frac{c}{2}} \zeta(m_1, m_2),
\]

\[
y(m_1, m_2) = \frac{1}{2} \left[ 1 + e^{-j 2 \pi \frac{m_1 \omega_1 + m_2 \omega_2}{N}} \right] \zeta(m_1, m_2)
\]

for \( m_1, m_2 = 0, 1, \ldots, N-1 \).

The corresponding DFTs of these functions are

\[
Y(k_1, k_2) = \left[ (1-e^{-j 2 \pi \frac{k_1}{N}})(1-e^{-j 2 \pi \frac{k_2}{N}}) \right]^{-\frac{c}{2}} W(k_1, k_2),
\]

\[
Y(k_1, k_2) = \left[ (1-e^{-j 2 \pi \frac{k_1}{N}})(1-e^{-j 2 \pi \frac{k_2}{N}}) \right]^{-\frac{c}{2}} W(k_1, k_2)
\]

where \( z_i \) is the delay operator associated with \( m_i \), \( \zeta(m_1, m_2) \) is an i.i.d. Gaussian sequence, and \( W(k_1, k_2) \) is the corresponding DFT.

2.1. Rotated And Projected Fractional Differencing Model

Note that this random function \( y(m_1, m_2) \) is defined on the surface normal plane because this model is based on a 2-D texture model.

To represent a 3-D rotated texture, we need two different sets of coordinate transformations. First one is the 2-D rotational coordinate transformation and second one is the orthographical projection coordinate transformation. The 2-D rotational coordinate transformation is needed to represent a texture image rotated on the 2-D image plane, and the orthographical coordinate transformation is needed to represent an orthographically projected texture surface onto the image plane due to the slant and tilt of texture surface.

Let \( \theta \) be the angle of rotation, \( \sigma \) the angle of slant, and \( T \) be the angle of tilt. Let \( Y''(k_1, k_2) \) represent the rotated and projected version of \( Y(k_1, k_2) \) where \( \eta_j'' = T_j k_1 + T_j k_2, j = 1, 2 \)

\[
Y''(k_1, k_2) = [(1-2\cos \omega_1 e^{-j 2 \pi \frac{k_1}{N}} + e^{-j 2 \pi \frac{\omega_1}{N}}) \times (1-2\cos \omega_2 e^{-j 2 \pi \frac{k_2}{N}} + e^{-j 2 \pi \frac{\omega_2}{N}})]^{-\frac{c}{2}} W''(k_1, k_2)
\]

where \( T_1, T_2, \) etc. involve the parameters \( \theta, \sigma, \) and \( T \). The details are available in [6].

2.2. Fractal Scale as a 3-D Rotational Invariant Parameter

Thus even with rotation and slanting, the transformed random texture function shares the same value of the fractal scaling parameter \( c \) with the original texture function \( y \). This invariance property is absent in the conventional 2-D stochastic model-based approaches such as AR model [7], facet model [8], etc.

3. Estimation of Parameters

The estimation of fractal scaling parameter \( c \) in the un-transformed first-order fractional differencing model (1) can be done by a simple least-square estimation scheme in the frequency domain based on a representation of the logarithm of the process which is linear in the parameters as follows. Let \( \alpha = -E[\log|W(k_1, k_2)|] \). \( \eta = (c, \alpha)^T \) can be estimated by minimizing the following cost function, which is quadratic in \( (c, \alpha) \).

\[
J(\eta, \omega_1, \omega_2) = \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} (\log|Y(k_1, k_2)| + c \frac{\pi k_1}{N} + \log|2\sin \frac{\pi k_2}{N}| + \alpha^2)
\]

\[
= \sum_{k_1=0}^{N-1} \sum_{k_2=0}^{N-1} (\log|Y(k_1, k_2)| - \eta^T Q(k_1, k_2))^2
\]

Here,

\[
Q(k_1, k_2) = \left[ -\frac{1}{2} (\log|2\sin \frac{\pi k_1}{N}| + \log|2\sin \frac{\pi k_2}{N}|) \right].
\]
Parameters \( \omega_1, \omega_2, \sigma, \tau, \theta \) in the projected second-order fractional differencing periodic model can be estimated in a similar manner using the DFT of (5).

4. Multi-level 3-D Rotational Invariant Classification Scheme

For this classification scheme, the images are separated into test and training sets. The class of textures and the number of classes in training set is assumed to be known \( \text{a priori} \).

4.1. The First-Level of Classification

In this level, the different 3-D rotated texture images are classified into the \( M \) different classes depending on their estimated values of the fractal scale. Actual classification is achieved by applying a distance classifier \( d(c,i) \), which measures a weighted distance between the extracted feature of test image denoted by \( \hat{c} \) and the mean feature of each of \( M \) classes. Then the texture is classified to class \( A_i \) for which such a distance is minimum. That is,

\[
d(c,i) = \frac{[\hat{c} - \bar{c}_i]^T}{\sum_{j=1}^{M} [\sigma_j^2]^{(i)}}
\]

and \( \bar{c}_i \) and \( [\sigma_j^2]^{(i)} \) correspond to the sample mean and variance of the feature \( c \) in class \( A_i \), respectively.

Then, it should be noticed that class \( A_i \) could consist of several different texture classes in the case that the different textures share the same fractal scale (the roughness of the surface), but have different patterns. This means that sometimes, checking the fractal scale only is not enough to distinguish the different patterns of texture. Thus, we need an additional classification scheme to distinguish textures even in the same class \( A_i \).

4.2. The Second-Level of Classification

In second-level, the textures which were already classified to the same class in first-level are split to the different sub-classes, based on the values of pattern features \( \omega_1, \omega_2 \) in the second-order fractional differencing periodic function (5).

\[
k^* = \text{minimum} d(\hat{c}^{(k)}, k), \quad k = 1, \ldots, N
\]

\[
d(\hat{c}^{(k)}, k) = \sum_{j=\omega_1, \omega_2} \frac{[f - f^{(k)}]^2}{\sum_{j=1}^{N} [\sigma_j^2]^{(j)}}
\]

and \( f^{(k)} \) and \( [\sigma_j^2]^{(k)} \) correspond to the sample mean and variance of subclass \( (k) \) features, respectively. Here, it should be noticed that since we have at most several subclasses from a first-level class, we need to compare only a small number of subclasses to complete the classification, instead of checking the feature distance of whole other texture classes. Thus, we can save the total processing time of classification by using this multi-level structure.

5. Experimental Results

For these experiments, nine different classes of texture were taken from Brodatz's standard texture album for the training set. These are, namely, grass[D9], tree bark[D12], straw[D15], herringbone weave[D17], woolen cloth[D19], calf leather[D24], beach sand[D29], water[D37], and raffia[D84]. Figure 1 shows the 256x256 original texture images of these.

For the actual training, sixteen \( 64 \times 64 \) sized sample image data were taken for each different texture pattern, and the sample mean and variance of parameters, \( c, \omega_1 \), and \( \omega_2 \) were obtained for each texture class, based on the first and second-order fractional differencing models [Table 1]. As we can see from Table 1, fractal scale \( c \) itself is not enough to classify the different textures, because some of textures have similar values of \( c \), even though they are different texture patterns. This is the reason why the classification will be completed in second level by considering additional parameter values of \( \omega_1, \omega_2 \). As mentioned before, since the estimation of \( c \) can be done by simple Least-square estimation and the estimation of \( \omega_1, \omega_2 \) can be done by Maximum Likelihood estimation, the two-level hierarchical classification structure induces the reasonably reduced processing time preserving the accuracy of the classification. Based on these sample mean and variance values, the number of classes for the first level of classification, \( M = 5 \), were determined. The following table [Table 2] shows the final classes for the first level of classification, and the corresponding sample mean and variance of each class. Notice that the herringbone weave texture belongs to the class 2 and 3, because of its high value of variance. Then, for the second level of classification, subclasses can be taken as the members.
of each class based on the different values of \( \omega_1 \) and \( \omega_2 \).

5.1. 2-D Rotated Texture Case

In this experiment, the test input images were taken from the 2-D raffia textures rotated by various angle \( \theta \). Then, each 64x64 texture was classified by the proposed multi-level classification scheme. For the first level, the fractal scale parameter \( c \) was extracted based on the first-order Fractional Differencing model (1), and the parameters, \( \omega_1 \) and \( \omega_2 \), were extracted from the second-order Fractional Differencing periodic model (5). Actual classification of the test images was done in each level by comparing weighted distances (6)-(9) between the extracted features and the data base. The classification results are presented in Table 3. Table 3 shows the parameter values extracted from each rotated texture pattern and demonstrates the perfect result of classification based on these values.

5.2. Rotated and Projected Texture Case

In this experiment, six 64x64 test input images were taken from the straw textures rotated and projected orthographically from the various tilted and slanted texture surface. Like in previous experiments, for the first level, the fractal scale parameter \( c \) was extracted based on the first-order Fractional Differencing model (1), and the parameters, \( \omega_1 \) and \( \omega_2 \), were extracted from the second-order Fractional Differencing periodic model (5). The classification results from this experiment are presented in Table 4. Table 4 shows the parameter values extracted from each rotated and projected texture pattern and demonstrates the perfect result of classification based on these values.

6. Conclusions

A new multi-level classification technique was proposed to classify the 3-D rotated texture surface. Unlike most classification schemes which have been suggested to date, this classification method can handle arbitrary 3-D rotated samples of textures, i.e., the accuracy of classification is not affected by the 3-D rotation of the test texture. The proposed multi-level classification scheme consists of two levels of classification procedure. In the first level of classification, a 3-D rotational invariant feature \( c \) (fractal scale) in the first-order Fractional Differencing model was extracted, and based on this value, the test data image was classified to a certain class. In the second level, each class was divided to the final desired subclasses based on two other texture pattern features, \( \omega_1 \) and \( \omega_2 \), which were extracted from the second-order Fractional Differencing periodic model. Then the input texture image was classified further in detail with these two pattern features. This multi-level classification has several advantages over the conventional one-level classification methods. First, since this algorithm uses a simple LS estimation to get the fractal scale \( c \) in first level, the processing time of the classification is reduced comparing to the methods that rely on only ML estimation. Second, from the additional texture patterns \( \omega_1 \), \( \omega_2 \) in second level, the more accurate classification can be achieved compared to the classification methods which have only one feature parameter \( c \). As a result of a series of experiments involving the differently oriented samples of natural textures, it is concluded that these combined features make possible for this multi-level classification method to have a strong class separability power for arbitrary oriented 3-D texture patterns.

A more detailed version of this paper is available from the first author.

References

Table 1: The sample mean and variance of parameter $c$, $\omega_1$, $\omega_2$: 16 64x64 sample image data are taken for each different texture class, and the parameter values are extracted from the first and second-order fractional differencing models (3.1-2).

<table>
<thead>
<tr>
<th>Textures</th>
<th>$c$</th>
<th>$\sigma^2$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>grass</td>
<td>1.209</td>
<td>0.057</td>
<td>0.744</td>
<td>0.078</td>
</tr>
<tr>
<td></td>
<td>1.530</td>
<td>0.073</td>
<td>0.691</td>
<td>0.199</td>
</tr>
<tr>
<td>straw</td>
<td>0.923</td>
<td>0.053</td>
<td>0.387</td>
<td>0.068</td>
</tr>
<tr>
<td>herringbone</td>
<td>1.003</td>
<td>0.072</td>
<td>1.263</td>
<td>0.114</td>
</tr>
<tr>
<td>weave</td>
<td>0.809</td>
<td>0.024</td>
<td>0.852</td>
<td>0.065</td>
</tr>
<tr>
<td>woolen cloth</td>
<td>1.064</td>
<td>0.044</td>
<td>1.175</td>
<td>0.114</td>
</tr>
<tr>
<td>calf leather</td>
<td>1.195</td>
<td>0.038</td>
<td>0.665</td>
<td>0.107</td>
</tr>
<tr>
<td>beach sand</td>
<td>1.074</td>
<td>0.055</td>
<td>0.083</td>
<td>0.064</td>
</tr>
<tr>
<td>water</td>
<td>1.547</td>
<td>0.062</td>
<td>1.042</td>
<td>0.153</td>
</tr>
<tr>
<td>raffia</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Database of the first level of classification. $\bar{\xi}$ and $\sigma^2$ are the sample mean and the variance of class $i$, respectively.

<table>
<thead>
<tr>
<th>Class</th>
<th>Textures</th>
<th>$\bar{\xi}$</th>
<th>$\sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>woolen cloth</td>
<td>0.809</td>
<td>0.024</td>
</tr>
<tr>
<td>2</td>
<td>straw, herringbone weave</td>
<td>0.963</td>
<td>0.063</td>
</tr>
<tr>
<td>3</td>
<td>herringbone weave, calf leather, water</td>
<td>1.047</td>
<td>0.055</td>
</tr>
<tr>
<td>4</td>
<td>grass, beach sand</td>
<td>1.202</td>
<td>0.045</td>
</tr>
<tr>
<td>5</td>
<td>tree bark, raffia</td>
<td>1.539</td>
<td>0.067</td>
</tr>
</tbody>
</table>

Table 3: Classification results from the 2-D rotated texture images. (Result indicates the result class after applying 2-level classification method.)

<table>
<thead>
<tr>
<th>Angles</th>
<th>$\bar{\xi}$</th>
<th>$\sigma^2$</th>
<th>$\omega_2$</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.532</td>
<td>1.132</td>
<td>1.098</td>
<td>raffia</td>
</tr>
<tr>
<td>40</td>
<td>1.517</td>
<td>1.144</td>
<td>1.102</td>
<td>raffia</td>
</tr>
<tr>
<td>60</td>
<td>1.535</td>
<td>1.138</td>
<td>1.119</td>
<td>raffia</td>
</tr>
<tr>
<td>80</td>
<td>1.537</td>
<td>1.142</td>
<td>1.120</td>
<td>raffia</td>
</tr>
<tr>
<td>100</td>
<td>1.532</td>
<td>1.139</td>
<td>1.118</td>
<td>raffia</td>
</tr>
<tr>
<td>120</td>
<td>1.529</td>
<td>1.138</td>
<td>1.120</td>
<td>raffia</td>
</tr>
<tr>
<td>140</td>
<td>1.527</td>
<td>1.135</td>
<td>1.107</td>
<td>raffia</td>
</tr>
<tr>
<td>160</td>
<td>1.533</td>
<td>1.140</td>
<td>1.113</td>
<td>raffia</td>
</tr>
<tr>
<td>180</td>
<td>1.525</td>
<td>1.133</td>
<td>1.099</td>
<td>raffia</td>
</tr>
</tbody>
</table>

Table 4: Classification results from the rotated and orthographically projected straw texture images. (Result indicates the result class after applying 2-level classification method.)

<table>
<thead>
<tr>
<th>Angles</th>
<th>$\bar{\xi}$</th>
<th>$\sigma^2$</th>
<th>$\omega_2$</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta = 0^\circ$, $\tau = 0^\circ$, $\sigma = 15^\circ$</td>
<td>0.914</td>
<td>0.585</td>
<td>1.189</td>
<td>straw</td>
</tr>
<tr>
<td>$\theta = 45^\circ$, $\tau = 0^\circ$, $\sigma = 30^\circ$</td>
<td>0.932</td>
<td>0.371</td>
<td>1.224</td>
<td>straw</td>
</tr>
<tr>
<td>$\theta = 90^\circ$, $\tau = 0^\circ$, $\sigma = 45^\circ$</td>
<td>0.928</td>
<td>0.373</td>
<td>1.218</td>
<td>straw</td>
</tr>
<tr>
<td>$\theta = 0^\circ$, $\tau = 45^\circ$, $\sigma = 15^\circ$</td>
<td>0.918</td>
<td>0.368</td>
<td>1.156</td>
<td>straw</td>
</tr>
<tr>
<td>$\theta = 45^\circ$, $\tau = 45^\circ$, $\sigma = 30^\circ$</td>
<td>0.922</td>
<td>0.375</td>
<td>1.191</td>
<td>straw</td>
</tr>
<tr>
<td>$\theta = 90^\circ$, $\tau = 45^\circ$, $\sigma = 45^\circ$</td>
<td>0.927</td>
<td>0.377</td>
<td>1.202</td>
<td>straw</td>
</tr>
</tbody>
</table>

Figure 1: 256x256 original texture images of training set: (a) grass (b) tree bark (c) straw (d) herringbone weave (e) woolen cloth (f) calf leather (g) beach sand (h) water (i) raffia

Figure 3: 64x64 straw texture images rotated and projected orthographically from the various tilted and slanted texture surface ($\theta, \tau, \sigma$ are the rotated, tilted and slanted angles): (a) $\theta = 0^\circ$, $\tau = 0^\circ$, $\sigma = 15^\circ$ (b) $\theta = 45^\circ$, $\tau = 0^\circ$, $\sigma = 30^\circ$ (c) $\theta = 0^\circ$, $\tau = 0^\circ$, $\sigma = 45^\circ$ (d) $\theta = 0^\circ$, $\tau = 45^\circ$, $\sigma = 15^\circ$ (e) $\theta = 45^\circ$, $\tau = 45^\circ$, $\sigma = 15^\circ$ (f) $\theta = 0^\circ$, $\tau = 45^\circ$, $\sigma = 15^\circ$ (g) $\theta = 45^\circ$, $\tau = 0^\circ$, $\sigma = 30^\circ$ (h) $\theta = 0^\circ$, $\tau = 0^\circ$, $\sigma = 45^\circ$ (i) $\theta = 90^\circ$